1 Introduction to Photonic Crystals and Photonic Band-Gaps

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Abstract. Photonic crystals continue to provide a fertile terrain for research effort around the world. Periodic refractive-index variation in two or three spatial dimensions, with a large high-low ratio produces a range of interesting phenomena that offer a variety of possibilities for device functionality. This chapter will introduce some of the basic concepts and the terminology required in order to understand the behaviour of photonic crystals.

Key words: Photonic Crystals, Photonic Band-gaps, Gap-Maps, Photonic crystal channel waveguides

1.1 Introduction

This chapter is concerned with the propagation of light in periodic structures, in particular structures that exhibit *refractive index* periodicity. It has become habitual (though not mandatory) for researchers in the field of optical propagation within and through such periodic structures to refer to them by the moniker '*photonic crystal*'. Indeed, photonic crystals have been a topic of major research interest since the seminal papers of Yablonovitch [1] and John [2] in 1987, although neither paper uses the term *photonic crystal*. Instead, the term was coined shortly after [3] the authors of these key papers had their first meeting, for lunch, in Princeton.

1.2 Brief Historical and Definitional Note

Photonic crystals are optical media with spatially periodic properties. However, this definition is too general to be useful in all contexts, and there has been some debate about the conditions under which it is legitimate to use the term [3]. We shall endeavour to by-pass this controversy simply by saying that if the medium of interest is predominantly periodic in either two or three space dimensions and there is a substantial contrast in the refractive index, it may be called a *photonic crystal*; though a further requirement is that at least one of the characteristic periods

in the medium is comparable with the wavelength of the light involved. A characteristic feature of wave propagation in spatially periodic media is the existence of stop-bands, i.e. regions of the frequency spectrum for which, in the ideal case, propagation is completely inhibited. The frequency range over which there is a stop-band along a specific direction depends on both the spatial periodicity along that direction and, where there is a spatially varying refractive index, on the refractive index *contrast*. At a single frequency, there is typically a finite angular range of directions over which the propagation is inhibited. Yablonovitch [1] realised that it might be possible for this angular range to be increased to cover the whole of three-space, given the correct distribution of the refractive index and a sufficiently large range of refractive index contrast. In that situation, it becomes appropriate to talk about a *photonic band-gap*, borrowing partly from the language of solid-state physics, and in particular the physics of *semiconductors*. From a historical perspective, it is interesting to note that the paper by Yablonovitch, which was published in 1987 [1], was anticipated to a substantial extent, and by more than a decade, in a short remark by Bykov [4], near the end of a paper concerned with the principles of distributed feedback (DFB) lasers.

1.3 What Does a Photonic Crystal Look Like?

Although unified by the possession of a periodic structure, which can clearly be seen in micrographs from a scanning electron microscope (SEM), photonic crystals now appear in a diverse range of forms. Even structures that are used in optical regimes, i.e. at wavelengths from the mid-infrared down to shorter wavelengths, extending as far as the ultra-violet, show a high level of diversity. In an SEM micrograph of a cleaved section of synthetic opal (Fig. 1.1), the intrinsic regularity of the square and hexagonal packing of the silica spheres from which opal is characteristically created can clearly be seen, giving the fundamental explanation of its properties. These structures are also present in naturally occurring opals, and therefore pre-date the epoch when mankind learned how to manufacture opal synthetically by millions of years. However, while natural opal often has additional components that

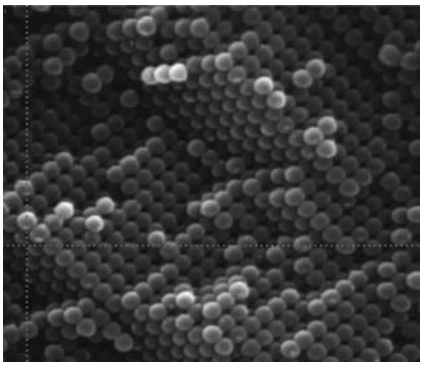

Fig. 1.1 Scanning electron micrograph of bare synthetic opal formed of sub-micrometre (typically ∼300 nm dia.) colloidal silica spheres. The cleaved surface shows both *square* packing associated with (100) planes and *hexagonal* packing associated with (111) planes

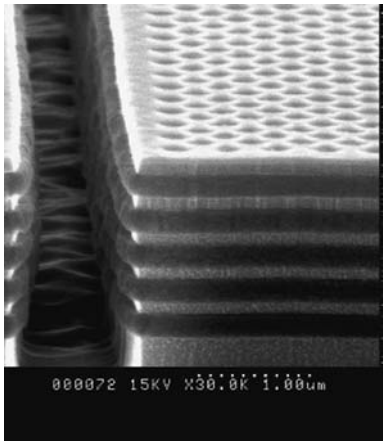

Fig. 1.2 Scanning Electron Micrograph of high refractive index contrast epitaxial 3D photonic crystal. Note the hexagonal array of air-holes etched into a GaAs/high-$x(Al_xGa_{1-x}As)$ DBR mirror stack that has been selectively oxidized

contribute to its cosmetic appearance and appeal, mankind has improved on this technology by controlling the conditions of the natural process. Because of its intrinsically invisible sub-wavelength scale structure, it was only with the advent of electron microscopy that images such as that presented in Fig. 1.1 could be obtained and, at the same time, provide the basis of a scientific explanation for the optical properties exhibited by opal.

Another example of a structure with a large periodic variation of the refractive index in all three spatial dimensions appears in Fig. 1.2. The general process by which the 3D photonic crystal structure shown in Fig. 1.2 has been produced differs considerably from that used to form synthetic opal [5]. The selectively oxidised structure shown is based on a periodically alternating multi-layer of material in the epitaxially grown GaAs/AlGaAs system. Fabrication of this 3D photonic crystal involves electron-beam lithographic (EBL) pattern definition and reactive ion etching (RIE) after the epitaxial growth stage, implying that the structure is formed by a 'top-down' process that is (within the precision limits of the process) deterministic. The subsequent selective oxidation process is also deterministic in that it is almost completely confined to the layers that contain a large aluminium fraction. The chemical composition and thickness of the alternating layers are also deterministic, although the precision is typically no better than one per cent, and the epitaxial growth process used involves an essentially open-loop form of control.

Nonetheless, there is an analogy with synthetic opal, at the atomic scale, in the formation of the above structure. Epitaxial growth of single crystal layers of semiconductors such as gallium arsenide (GaAs) or aluminium gallium arsenide (AlGaAs) involves self-organisation, with the gallium, aluminium and arsenic atoms coming together in a ratio (between the *total* number of group III atoms {Al and Ga} and the total number of group V atoms {As}) that is very close to 1:1. These atoms bind into regular structures that may contain millions of atoms, but have no discernible deviation from perfect spatial periodicity.

1.4 Photonic Band-Structure and the Electron–Photon Analogy

We have already introduced the terms 'stop-band' and 'band-gap' to describe the properties of photonic crystals. We shall now extend the comparison between photonic crystals and semiconductors, and indeed expose its limitations. The propagation of electromagnetic waves in media with a periodic spatial variation of properties, such as the refractive index, leads to the existence of *band-structure*. Single crystal solids are inherently anisotropic, and the corresponding electronic band-structure reflects this anisotropy. The standard way to represent band-structure, in the context of the electronic properties of solids (e.g., single-crystal sections of semiconducting materials such as silicon, and the III–V compounds gallium arsenide (GaAs) and indium phosphide (InP)), is to generate a graph of energy *vs.* momentum. Because real matter has three space dimensions and we are plotting energy *vs.* momentum as it occurs in 'three-space', graphical representations that can easily be assimilated by the human eye, despite being presented on a 2D surface, are essentially pseudo-3D images that rely on the nature of human perception for their effectiveness. Curves of energy are plotted against momentum along a specified direction, in relation to the axes defining the crystal lattice, then multiple sections are incorporated through rotation about the three principal axes, the orientation of which is determined by the symmetry of the crystal.

Although conductivity in single-crystal semiconductors may exhibit anisotropy, this anisotropy is typically unimportant and ignored in the most widely used applications of semiconductors. In the characteristic situation where a controlled amount of current is passed through part of a semiconductor device, such as a conventional metal oxide semiconductor (MOS) or bi-polar transistor, the electron or hole transport is *incoherent*. The flow of the *drift* current is determined by the acceleration of the electrons or holes that results from the existence of a local electric field, in combination with an almost immediate scattering or collision event that routinely involves lattice vibrations or phonons. The resulting, temperature-dependent, property of the semiconductor is called the *mobility*, and there is a linear relationship between this and the *conductivity*. In many cases, e.g. that of the transport of current through a p-n junction, the existence of a carrier concentration gradient leads to current flow by *diffusion*, but the situation is still typically mediated by phonons, as is implied by the direct, linear relationship between the diffusion constant and the mobility. Therefore, even though it is governed by wave mechanics (i.e., quantum mechanics), the nature of electronic conduction is typically diffusive. However, this is radically different from the situation sought and exploited when propagating light (photons) through photonic crystals.

Photonic crystal structures and their corresponding band-structure are both significantly different from and closely analogous with semiconductors. If the photonic structure that is being investigated or exploited is sufficiently regular (i.e., if the structure is close to perfect), the scattering of light, other than the coherent scattering that leads to the existence of band-structure (i.e., Bragg-diffraction writ large), should be as small as possible. The propagation of optical frequency electromagnetic waves through a photonic crystal typically shows strongly frequency-

dependent behaviour, but is nevertheless intrinsically loss-less in the absence of imperfections and absorption. The ability of a monochromatic source of light to launch light into, or from within, a photonic crystal is determined by its detailed characteristics and by the properties of the photonic crystal at the optical frequency of interest. In general, it is not possible to match the source and the photonic crystal structure perfectly and simultaneously over a range of frequencies, even if perfect matching can be obtained at one or more specific frequencies. This unavoidable connection is a characteristic aspect of Maxwell's equations, but it also has quantum consequences. In particular, when considering the amount of light that can be transmitted through a photonic crystal device structure with a light source placed *outside* the structure, there may be a substantial level of reflection, with the level of power reflected being strongly dependent on the optical frequency. In the absence of losses, power conservation implies that light is either transmitted into the photonic crystal, possibly exciting optical wave propagation that is distributed over several modes of the photonic crystal, or that light is 'reflected' by the crystal. In this context, the reflected light includes both light that is specularly reflected by the crystal, and light that is diffracted by the crystal back into the half-space within which it is incident. The direction in which light is diffracted is determined by the photonic crystal lattice periodicities, while the amount of diffracted light is determined by the refractive index distribution in the photonic crystal, as seen through the defined surface of the photonic crystal. Power and energy conservation mean that the sum total of the power in the transmitted, reflected and coherently diffracted optical beams equals that launched into the system.

Therefore, despite the differences between photonic crystals and semiconductors, a direct conceptual transition between the descriptions of 'electronic' and 'photonic' band structure is possible – assuming the immediate conversion, justified by quantum concepts, between energy and frequency – and between momentum and propagation constant.

1.5 Propagation of Light in Periodic Media, Photonic Crystals and Photonic Band-Gaps

Solutions of Maxwell's Equations for propagation in loss-less periodic dielectric media are themselves periodic, and thus provide the Bloch modes or Floquet–Bloch modes of the periodic structure (as explored by Joannopoulos and co-authors [6]). The Bloch modes for a photonic crystal structure at a single frequency are described by the product of a complex exponential propagation term (for each axis along which there is periodicity), multiplied by a term that has exactly the periodicity of the photonic crystal structure, i.e., where $u(x,y)$ is a function that has the periodicities of the photonic crystal, the solution has the form (in 2-space):

$$\exp(\mathrm{j}k_x x) \times \exp(\mathrm{j}k_y y) \times u(x,y) \times \exp(\mathrm{j}\omega t) \ . \tag{1.1}$$

The usual convention applies of taking the real part of the whole complex solution to obtain the instantaneous field quantities. This solution has a discrete, two-

dimensionally periodic, Fourier-spectrum in reciprocal space (or k-space) for a 2D photonic crystal. The equation immediately below this paragraph is a formal representation of a Bloch mode in a 2D periodic lattice as a summation of space harmonics. The weighting factors $h_{n,m(k)}$ are the Fourier coefficients and $G_{n,m}$ are the reciprocal lattice vectors. The magnitude and phase of the space harmonic components forming the Bloch mode depend on the details of the periodic function specifying the refractive index distribution of the photonic crystal. In our 3D-Universe, the function $u(x,y)$ will also have a z-dependence that is determined by the detailed 'vertical' distribution of the 2D photonic crystal structure, e.g., a waveguide layer with holes in it, typically supported by a substrate that has a refractive index greater than that of air, but less than that of the waveguide layer:

$$H_k(\bar{r}) = \sum_{n,m} h_{n,m(k)} H_0 \exp\left[\mathrm{i}\left(\bar{k} + \bar{G}_{n,m}\right)\bar{r}\right] \ . \tag{1.2}$$

Bloch waves are typically partial standing waves – and can therefore transport optical power. However, they become purely standing waves that transport no power at the Brillouin-zone boundaries, i.e. at the band-edges, where the slope of the ω–k diagram goes to zero and the group velocity is therefore also zero. As the Brillouin-zone boundary is approached, both the *magnitude* of the slope of the dispersion curve for the Bloch mode and the group velocity decrease progressively towards zero. In a typical situation, the shape of the Bloch-mode dispersion curve is parabolic at the zone-boundary and the curve is, as already implied, exactly horizontal. Although time does not permit us to examine the detail, the reader may be interested to read the papers by Lombardet and co-workers [7, 8]. In these papers, calculations are described in which the group velocity is calculated via summation, overall all of the Brillouin zones, of the weighted space-harmonic contributions to the total Poynting-vector of the Bloch wave, divided (for normalisation purposes) by the energy density. Although direct estimation of the group velocity as the gradient of the ω–k surface may well be a simpler process, the space-harmonic Poynting-vector summation process could be a useful tool in solving more complex problems that involve the presence of gain, absorption and non-linearity.

For wave propagation in a dispersive situation it is typical to consider both the *phase-velocity*, v_p, defined as the ratio, at any point on the dispersion curve for a particular mode of propagation of the angular frequency (in radians per second) to the propagation constant; and the *group velocity*, v_g, defined as the local gradient of the dispersion curve that relates photon energy to photon momentum. Multiplying Planck's constant by the cyclic frequency or the wave propagation constant, respectively, gives the corresponding energy and momentum. Although it is natural to think of the frequency as being the independent variable and the propagation constant as being the dependent one, the standard convention is nevertheless to display the graph of dispersion with the angular frequency forming the vertical axis and the propagation constant forming the horizontal axis.

In a medium that has a periodically distributed refractive index (in 'real' space), the propagation of the electronic/optical waves also exhibits periodicity in the space of possible propagation constant values ('k-space'). This periodicity in k-space is

well understood and leads to the definition of Brillouin zones. For a crystal with lattice constant a, the relevant wave number that defines the Brillouin zone boundaries is $2\pi/a$. In addition, the size and shape of the first Brillouin zone in a two-dimensionally periodic crystal depend on both the lattice constants and on the crystal symmetry. For simple lattices, such as those with square or hexagonal symmetry, there is a single lattice constant that is just the distance between all nearest neighbour lattice points.

Because of the periodic nature of the medium, the allowed solutions of the appropriate wave equation are Bloch waves. Bloch waves are made up of space-harmonics – and the consequence is that there is not a single unique phase velocity that describes the propagation of the Bloch wave. In contrast, the group velocity is defined by the slope of the dispersion surface, which is unique, i.e. the same value is obtained for the group velocity at any point that is equivalent between Brillouin zones. Bloch modes at a single frequency and along a specified propagation direction typically have both forward and reverse travelling space-harmonic components. Situations where the space-harmonic components in corresponding negative and positive parts of the Brillouin zones all have equal magnitude are possible – and there is then no net transport of energy. Such situations are clearly pure standing-wave situations, and the following Bragg condition applies:

$$k = k_{\mathrm{B}} = \pm(\pi/a) \times (\text{non-zero integer}) \ . \tag{1.3}$$

At the Brilloun-zone boundary, there is a region of frequencies where the dispersion curves are separated by a gap, i.e. there is a band-gap region in which propagation along the specified direction is forbidden. It is usual to talk about the stop-band edges and to label the upper frequency limit of the stop-band as the conduction-band and the lower frequency limit as the valence band. Although these terms only have an immediate physical meaning for the electron probability waves of the solid-state physical theory of semiconductors, they are nonetheless also used for optical frequency electromagnetic waves. On the other hand, for electromagnetic wave propagation in regular arrays of high dielectric constant pillars in air or air-holes in a high dielectric constant background, the bands are more usefully labelled as the air-band and the dielectric band, respectively. The two distinct Bloch modes that can be supported by the photonic crystal may have the same value for their k-vector, but their modal power/energy distributions, and the resulting average energies in each mode, are quite different.

1.6 Quantum Energy–Frequency Relations

In quantum language, the photon energy E (Joules) and the optical frequency f or ν are related by:

$$E = hf = h\nu = (h/2\pi)\omega = \hbar\omega \ . \tag{1.4}$$

In this equation, f and ν are alternative symbolic representations of the cyclic frequency in Hertz, ω is the angular frequency in radian/second, and h is Planck's

constant ($\sim 6.6 \times 10^{-34}$ Joules/second). Again using quantum language, there is the relationship between momentum, p, and propagation constant, k, that:

$$p = \hbar k \, . \tag{1.5}$$

Plots (in principle) in three-space of the relationship between ω and k describe the properties of a medium from the point of view of wave propagation. For periodic media, there are specific repetitive relationships that are important for an understanding of the properties of the medium. Given an ω–k diagram, it is the ratio of the value of ω along the y-axis to the value of k along the x-axis, in a plot of ω versus k, that gives the *phase* velocity, i.e.:

$$v_{\mathrm{p}} = \omega / k \, , \tag{1.6}$$

where ω is the angular frequency (in radians per second) and k is the wave vector magnitude, which has dimensions of inverse length. *Group* velocity is then defined as the *slope* of the dispersion curve, i.e.:

$$v_{\mathrm{g}} = \partial \omega / \partial k \, . \tag{1.7}$$

Under conditions of so-called 'normal dispersion' it is usually legitimate to associate the group velocity with the *energy* velocity. But group velocity and phase velocity are both *vectorial* properties of a medium (in three-space). So the group velocity definition should be understood as implying the use of the *gradient operator* from vector calculus.

In considering the propagation of optical waves in media such as photonic crystals, it is often useful to use the free-space ('air') *light-line* as a reference marker – or, alternatively, the *substrate* light-line (e.g. when considering situations such as 2D photonic crystals fabricated in planar waveguides). The light line is a straight line through the origin of an ω–k diagram. The slope of the light line for a given medium is simply given by c/n, where n is the refractive index of the medium. For free-space, the slope of the light line is c, i.e. $3 \times 10^8 \, \mathrm{m\,s^{-1}}$. In typical semiconductor media such as silicon and gallium arsenide, the slope is more than three times smaller, i.e. below $10^8 \, \mathrm{m\,s^{-1}}$. For the layered open waveguide photonic crystal structures that are used in much of photonic crystal device research, the strictly guided Bloch modes must have an effective refractive index that is larger than the external media refractive indices. So the guided Bloch mode dispersion curves must lie *below all* the relevant light-lines. The term '*heavy photon*' has been coined [9] for situations where the group velocity is significantly smaller than the modal phase velocity – and is based on the standard association of particle-like behaviour in wave mechanics with the group-velocity of the waves. However, it is not appropriate to assign mass to such heavy photons. Instead, one may talk about photons that have modified values for their momentum.

1.7 Light Confinement in One Spatial Dimension: The Slab Waveguide

Let us now consider the simple planar slab waveguide, which has a uniformly thick core layer occupying the whole of 2-space and having a characteristic thickness a in the y-direction, i.e. the direction of the normal to the two surfaces that define the slab. The simplest case, but one that is of practical relevance, is a dielectric slab with a refractive index (n_{slab}) that is greater than one, with air (refractive index $n = 1$) above and below the slab (Fig. 1.3). The high refractive index dielectric slab waveguide is a simple example of a structure that provides strong confinement for light. For high refractive index values at optical frequencies (e.g. refractive index $n = 3.5$), the slab has the desirable property of supporting only a single fully confined guided mode of each polarisation type (see below) at thicknesses up to approximately 0.3 μm.

The dispersion equation that describes the propagation of strictly guided electromagnetic waves in an infinite dielectric slab, with lower refractive index media above and below it, is based on solution forms for the field distributions of the discrete mode that have simple trigonometric function dependence in the slab (i.e. the waveguide core) and exponential decay outside. Boundary conditions in the form of continuity of parallel field components across the slab boundaries lead to the dispersion equations for two distinct classes of modes, typically characterised in engineering electromagnetic notation as transverse electric (TE) and transverse magnetic (TM) modes. TE modes have no electric field component parallel to the propagation direction and TM modes have no magnetic field component parallel to the propagation direction. Of greater physical importance, is the fact that the single *electric* field component in the TE modes is characteristically normal to the propagation direction and parallel to the surfaces of the slab, while for the TM modes, the single *magnetic* field component is parallel to the surfaces of the slab. Correspondingly, the main (but not the only) magnetic field component of the TE modes is oriented normally to the

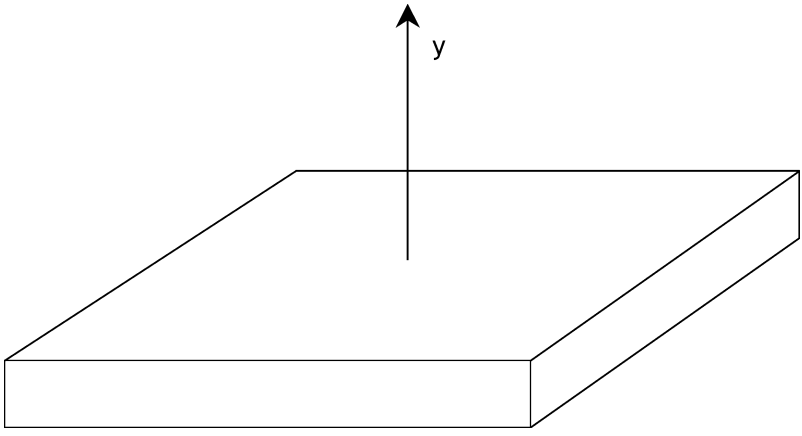

Fig. 1.3 Simple dielectric slab optical (electromagnetic) waveguide

surfaces of the slab, while the main (but not the only) electric field component of the TM modes is normal to the surfaces of the slab. The convention just stated is that used in Joannopoulos et al. [6], as well as the standard electromagnetic engineering textbooks. Unfortunately this is not a universally applied convention, and care must be taken when using popular software packages for modelling of photonic crystal structures, since at least one such package has used exactly the opposite convention!

Although the functions used to describe the modal field distributions for the dielectric slab waveguide are simple trigonometric ones, and the resulting dispersion equation is quite compact, it is a transcendental equation. Therefore the solutions for the propagation constants of the guided modes at a given slab thickness, with given refractive indices and with given wave frequency, have to be found numerically, i.e. by a simple root-solving process. In contrast, it is useful to note that specification of the wave frequency and the propagation constant value can be used, with the same equation, to compute multiple waveguide thicknesses that correspond to the successive modes of the waveguide. (The equation is not a transcendental one as far as the waveguide thickness is concerned.)

An important practical point is that thin slab waveguides are the base for the air-suspended high-index membranes that have been used in many of the 2D photonic crystal structures described in the recent literature, in particular for the realisation of high Q-factor photonic crystal micro-cavities. The weighted average refractive index of a thin dielectric slab with a regular pattern of air holes in it can readily be much larger than that of air, making it possible to support guided and well-confined Bloch modes with a range of effective refractive index values. In particular, the effective guided mode Bloch index can have a value as large as 3, making it possible for full photonic band-gap behaviour to be obtained for in-plane guided propagation.

The mode structure of a uniform dielectric slab that is symmetrically enclosed by half spaces having equal refractive index is shown schematically in Figure 1.4. The vertical axis is a dimensionless normalised frequency, $(\omega a/2\pi c) = a/\lambda_0$, and the horizontal axis is a normalised in-plane wave-vector or propagation constant, $(ka/2\pi) = (a/\lambda_{\text{eff}})$. The fundamental mode of the symmetric slab exists, for a given frequency, down to zero thickness. This property is closely related to that of the fundamental mode of a circular fibre, which also does not exhibit a low-frequency cut-off. In the *asymmetric* situation, e.g. of a thin layer of silicon with air above and supported by a sufficiently thick layer of silica (refractive index $n = 1.45$), the lowest order mode at a given wavelength, e.g. the free-space wavelength of 1.55 μm that is widely used in fibre-optical telecommunications, is cut-off below a finite thickness value that is determined by the refractive index of the slab. The cut-off thickness, at a given wavelength, is slightly different for a TE mode and its corresponding TM mode, implying that there is a narrow range of conditions for which only the lowest order TE mode is supported by the asymmetric slab waveguide. The vertical confinement is characteristically weak for this specific condition.

There are two distinct regions that should be noted in Fig. 1.4. The guided modes (identified with the label n) form a discrete spectrum that always lies between a steeper straight line that corresponds to the velocity of light in air or, to a good approximation, in vacuum, and a less steep straight line that corresponds to

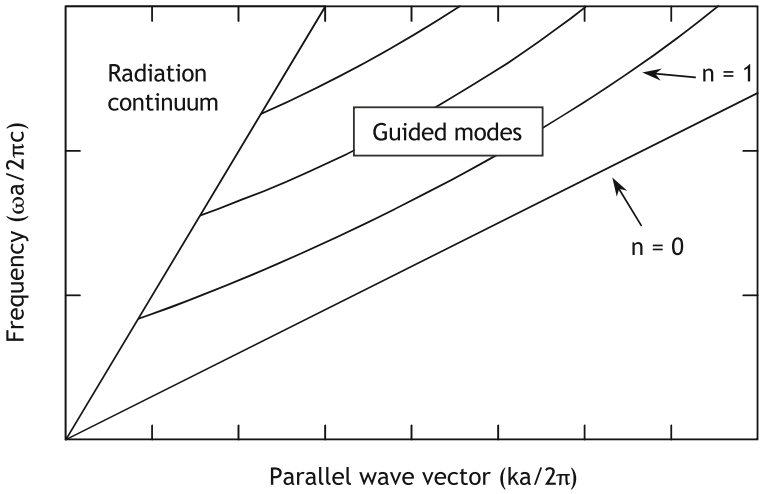

Fig. 1.4 Schematic mode structure of dielectric slab waveguide

the velocity of plane waves in the unconfined (bulk) dielectric medium. (The relatively modest contribution of material dispersion is ignored in this diagram.) The permitted states that are labelled as 'radiation continuum' in Fig. 1.4 have phase velocities that are greater than the free-space velocity of light, and are therefore radiative modes of the slab. Unlike the guided-modes, which have a discrete spectrum, the radiative modes have a continuous spectrum. They may have any phase velocity (along the guide) that is greater than the velocity of light in free-space. If a plane wave is launched in air onto a dielectric slab at an angle that corresponds to an extended state, the wave will be partially transmitted and partially reflected by the slab, emerging at the same angle as the angle of incidence, if the slab has air on both sides. That this behaviour is nevertheless characteristically modal in nature is shown by the fact that the amount of reflected and transmitted light depends on both the angle of incidence and the thickness of the slab, relative to the free-space wavelength and the refractive index of the slab. With the correct choice of parameters, the coupling between a free-space beam and the modes in a photonic crystal structure embedded in a slab waveguide may be sharply resonant in nature, corresponding to large and complementary variations in reflection and transmission.

The partial description that we have given of the dielectric slab modal properties connects back to classic papers on dielectric waveguides, such as that of Marcuse [10], which discussed the propagation losses resulting from a specified level of roughness. In the context of 1D or 2D photonic crystal structures composed of holes etched into slab waveguides, the earlier work on the scattering of light (mainly into radiation modes) due to roughness in a slab waveguide provides a useful starting point, with the roughness treated by Marcuse (and in subsequent work by Payne and Lacey [11]) now being replaced by structural aspects of the photonic crystal. In passing, we may note that the scattering of light due to roughness and irregularity is

a continuing concern in the evolution of nano-photonic waveguides such as photonic wires [12] and photonic crystal channel guides [13].

Experiments designed to estimate the characteristic band-structure of planar photonic crystal structures [14, 15] frequently use finite (but large enough) cross-sectional area light beams incident at known angles on the photonic crystal region. Measurements of the reflection and transmission coefficients as a function of frequency then enable identification of the free-space coupled or leaky Bloch modes of the planar photonic crystal. Such measurements are subsequently used to construct the band-structure of the photonic crystal. This technique can be extended to allow identification and characterisation of the guided Bloch modes, via the use of the prism-coupler technique or by illuminating the edge of a finite area photonic crystal region with the probe light beam [16]. The angle of incidence of the light beam immediately identifies the in-plane propagation constant (wave-vector), while rotation of the planar photonic crystal structure around an axis that is normal to the plane of the crystal identifies the direction of propagation with respect to the defining axes of the photonic crystal lattice. The polarisation of the incident light beam must be specified, and the polarisation content of the reflected, transmitted and diffracted light beams must be measured.

Another nomenclature-related discrepancy in the analysis of the guided modes of a dielectric slab is that one standard convention labels the lowest order, strictly-guided, mode as $n = 0$. This is because it associates n with the multiple solutions of the dispersion equation, which are in turn directly associated with the multiple branches of the *tan* and *cotan* functions – and the multiple angles that give the same values of the tan and cotan functions. Our convention here is *not* the one used in Joannopoulos et al. [6].

1.8 Band-Structure in Periodic Media, the 1D Case

Despite the desire of some researchers in the field to restrict the use of the words 'photonic crystal' to media that are periodic in either two or three spatial dimensions, there is much to be gained from understanding the behaviour of optical waves in media that are only periodic along one spatial axis. In this 1D situation, the periodic variation of the electromagnetic properties, e.g. the refractive index, guarantees that there will be band structure, i.e. there will be curves in ω–k space that describe the propagation of electromagnetic waves over a specified range of photon energies, i.e. optical wave frequencies, despite the spacing between the curves implying that propagation is not possible (in the ideal case of a perfect, infinitely repeating lattice).

The representative band structure that can be observed in or calculated for a one-dimensionally periodic dielectric structure with large refractive index contrast between the two constituent layers that form a single spatial period is shown in Fig. 1.5. This pattern is, in principle, merely the first section of a graph that is repeated infinitely-many times along the horizontal (wave-vector) axis in both the positive and the negative directions. The diagram is therefore a representation of the electromagnetic wave dispersion of the periodic system in the first Brillouin zone. Note,

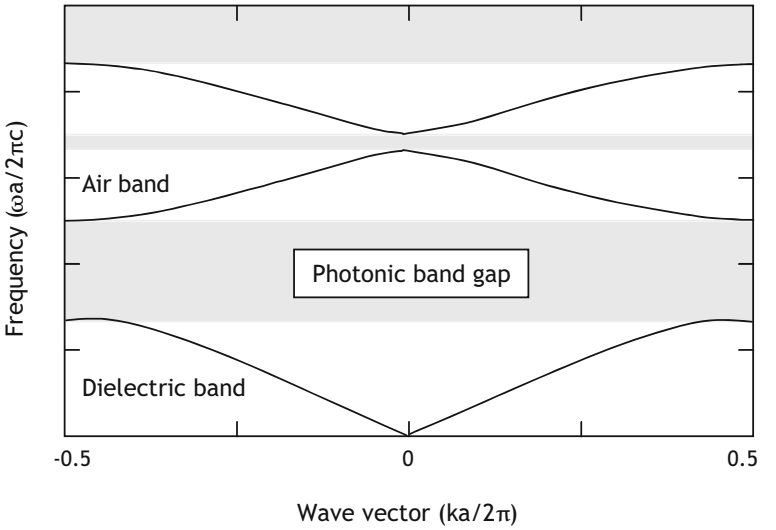

Fig. 1.5 Schematic dispersion (ω–k) diagram of large refractive-index contrast 1D periodic structure

again, that the dispersion curves describing the band structure must intersect the Brillouin-zone edges exactly horizontally.

Brillouin zones are readily defined in ω–k space, even for situations that are only periodic in one space dimension. All Brillouin zones have the same extent along the k-axis, because it is possible to add on a reciprocal lattice vector and obtain a solution that is formally indistinguishable, i.e. unchanged. The Floquet–Bloch modes for a particular lattice are characteristic of that lattice, as their behaviour is determined by the detailed properties of the lattice. These properties include the exact refractive index distribution for optical waves, or the potential distribution for electron waves.

Built-in to the band-structure diagram of a periodic medium are the possibilities of both slow-wave propagation and backward waves. It is important for all researchers currently working in the field of slow light phenomena (and slow electromagnetic wave propagation more generally) to have a proper awareness of key historical aspects of their field. A good introduction to this is found as the web-site:

http://en.wikipedia.org/wiki/Backward_wave_oscillator

It has long been recognised that slow electromagnetic-wave-propagation behaviour on periodic electrode structures could be used as a means of extracting microwave frequency electromagnetic power from a high velocity electron beam in a vacuum, typically with the periodic electrode structure wrapped helically around a cylindrical electron beam. In Amerenglish, devices that exploit this interaction are known as *microwave tubes*. With appropriate construction and choice of parameters, this interaction between electromagnetic waves and electron beam charge oscillations can produce backward wave oscillation, in which the group and phase velocity of the electromagnetic waves have opposite signs. Feedback mechanisms then allow

the classic build-up from noise into a characteristic oscillation. A key aspect of travelling-wave type microwave tubes, which may yet find a modern counterpart at optical frequencies, is that the wave propagation is radically modified from that of Bloch waves in a loss-less, gain-free medium. In microwave frequency electron-tubes, the presence of gain enables a useful interaction to take place that involves only a single space harmonic of the periodic structure, i.e. all the important action may be crowded into a single Brillouin zone. Present day microwave tubes are able to operate at frequencies up to 1 THz.

1.9 Band-Structure in Periodic Media, the 2D Case

Moving to photonic crystals with two-dimensionally (2D) periodic lattices radically increases the scope for engineering of the photonic band-structure. A variety of lattice symmetries becomes available, of which the 'triangular' lattice shown schematically in Fig. 1.6 is merely the simplest example. The triangular lattice is so called because the smallest group of holes that defines it has a triangular arrangement. But the lattice has six-fold rotationally symmetry that allows the circular building blocks, in principle, to be close-packed, so it may also be described as 'hexagonally close-packed'.

Included among the possible arrangements of interest for creating photonic crystal structures in two-space are: (i) rectangular lattices with different periodicities along two orthogonal axes; (ii) photonic quasi crystal structures that are strictly not periodic but have rotational symmetries higher than six; and, (iii) 'graphite' lattices that also have six-fold rotational symmetry, but are more open than the triangular lattice and therefore cannot be close-packed.

Practical 2D photonic crystal structures are typically based on planar waveguides, in order to provide confinement along the third spatial dimension. The use

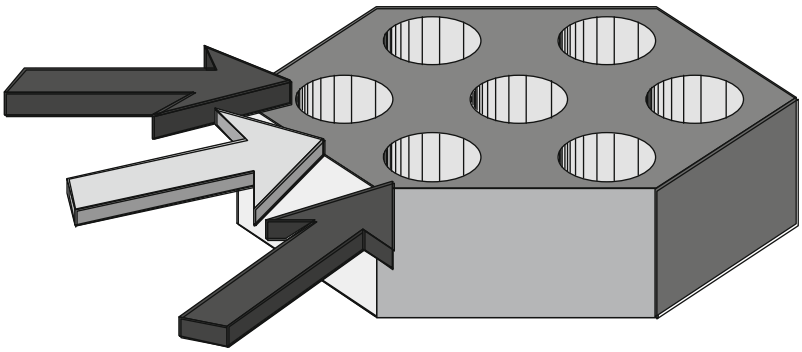

Fig. 1.6 Schematic of section of a 'triangular' photonic crystal lattice of circular air holes in a solid medium. The section has been chosen in order to emphasise the hexagonally close packed symmetry of the lattice. The *two darker arrows* indicate symmetrically equivalent directions, while the *lighter arrow* indicates a direction that exactly is half-way between

of air holes in a continuous high refractive-index medium, rather than high-index pillars in a low index (air) background, provides much better 'vertical' (waveguide) confinement, as well as simple mechanical integrity. The waveguide used may, depending on circumstances, take the form of a suspended membrane in air, e.g. of silicon or an epitaxial III–V semiconductor structure, of a high index core layer on a low index cladding layer, e.g. SOI (silicon-on-insulator) or a low-contrast, but high-index waveguide layer, with holes etched to a sufficient depth, 'from the top', through the waveguide core and into the lower cladding layer.

In contrast to the hexagonal (triangular) lattice, a square photonic crystal lattice has the simple advantage, both for computational modelling purposes and for pattern definition purposes, of being easier to specify in a rectilinear coordinate geometry. For some purposes, the greater degree of anisotropy, i.e. the smaller symmetry level, of the square lattice may even be a useful feature.

Figure 1.7 shows schematically the band-structure of a particular triangular lattice 2D photonic crystal structure. The parameters of the crystal, such as the refractive indices and the hole filling-factor, have been chosen so that there is a full photonic band-gap (PBG), i.e. (as indicated) there is a range of energies where no propagation band is available. As the figure suggests and as will emerge in subsequent chapters of this book, exploiting fully the potential of a band-structure as complex as that which is automatically provided by a 2D periodic lattice of strong light scatterers is a serious challenge – and is an on-going one in research, world-wide. Even if there is no region where a full photonic band-gap or a substantial angular range of photonic band-gap behaviour, many interesting properties become available, e.g. slow-propagation, super-prism, hyper-lens behaviour and auto-collimation, strongly polarisation-dependent effects and largely polarisation-

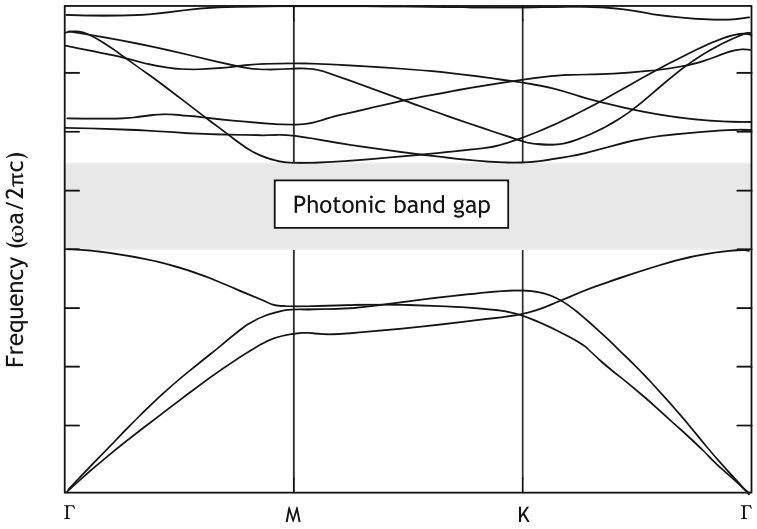

Fig. 1.7 Schematic of band-structure for 2D lattice

independent effects. Changes in the band-structure and the introduction of useful 'states-in-the band-gap' add to the possibilities.

Figure 1.7 follows standard conventions [6] in terms of showing the dispersion along the two high-symmetry directions of a hexagonal photonic crystal lattice, together with the change in the frequency (or, equivalently, photon energy) encountered in moving along the Brillouin-zone boundary between the two high-symmetry directions. Effectively this representation is made up from three chosen cross-sections of the frequency surface in reciprocal lattice space.

1.10 Gap Maps

Gap maps [17] are a representational tool that show graphically, for a given lattice form and a given combination of the high and low refractive-indices of a two material based photonic crystal, regions of the spectrum over which the polarisation-determined phenomenon of 'full' photonic band-gap ('full-PBG') behaviour occurs. These are the regions where propagation in any direction encounters a stop-band. The low index medium involved is usually air, but this is not a mandatory condition. The other variable involved, for circular holes in a high-index solid medium or for high-index pillars in air, is the ratio of the cylinder radius r to the lattice constant a, i.e. the normalised radius. This ratio is directly related to the filling factor, which is defined as the fraction of the total area of the 2D lattice (in plan view) that is occupied by the cylinders. The regions where fully polarisation independent PBG behaviour is obtained simultaneously for both TE and TM polarisations can then

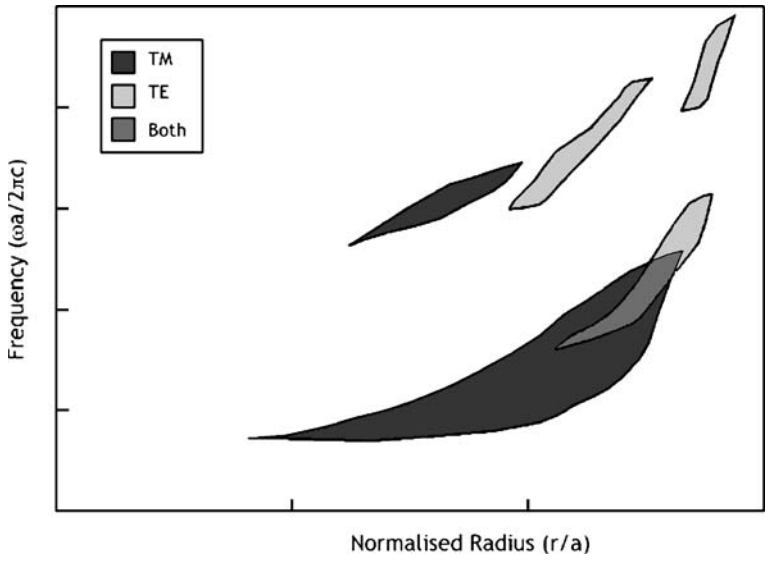

Fig. 1.8 Schematic gap-map

be identified as the characteristically much smaller overlap zones between the PBG regions for the two different polarisations. As Fig. 1.8 shows, such overlap, which may be very desirable for some applications, does not always occur. The other side of the coin, polarisation selectivity, is clearly also potentially useful.

1.11 Channel Waveguides Through Photonic Crystals

The presence of deliberately induced 'defects' in a photonic crystal region is an essential tool for their exploitation in device structures. Associated with such defects, in a manner closely analogous to the effects of doping on the electronic properties of semiconductors, is the creation of 'states in the band-gap'. One specific type of defect of interest is the channel waveguide, which may be obtained, for example, by filling in one row of holes in a photonic crystal lattice – forming a W1 channel, as illustrated in Fig. 1.9. But channel waveguides can also be formed, conceptually, by moving blocks of crystal, making it possible to produce channel waveguides with fractional spacing, e.g. W0.7 or W1.3.

Figure 1.10 shows a possible dispersion curve for a photonic crystal channel wave waveguide, set in the framework of the first Brillouin zone along the main propagation axis and the band structure of the perfectly regular lattice. Although only a single modal dispersion curve is shown in the band-gap region of the band-structure, in general there will be a discrete set of guided modes. The guided-mode dispersion curve is not shown as continuing above the upper band beyond the point where it intersects with it, but this is possible. The guided modal dispersion curve is also shown as continuing for k-vectors down to zero, corresponding to an infinite phase velocity. At the same time, the dispersion curve must be exactly horizontal at its intersection with the frequency axis, corresponding to zero group velocity, with slow propagation behaviour occurring in this neighbourhood.

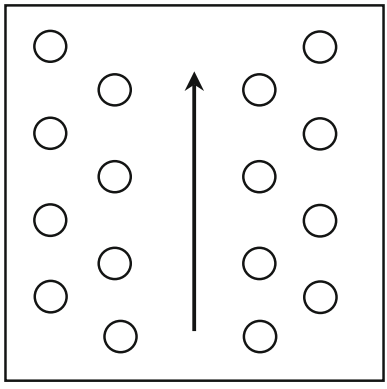

Fig. 1.9 Schematic of W1 channel guide in triangular (hexagonal) lattice of cylinders

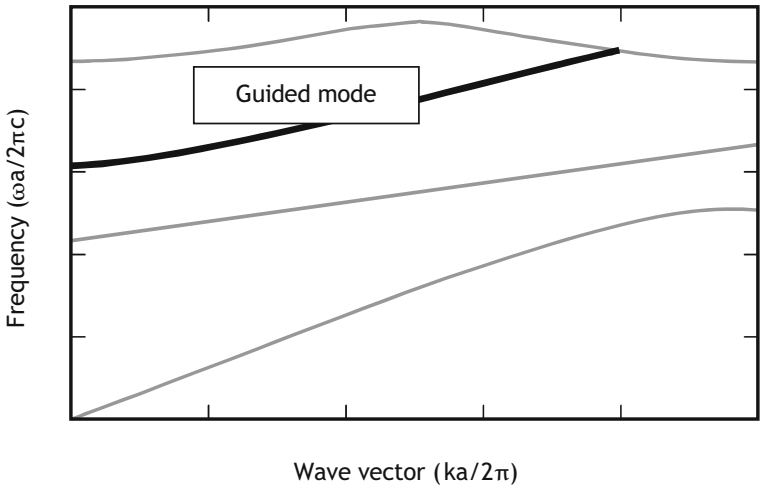

Fig. 1.10 Photonic crystal band-structure for lattice with channel-waveguide type defect state

1.12 Concluding Remarks

This introductory chapter has provided a brief overview of the topic of photonic crystals and the concept of the photonic band-gap; however, there are many more aspects of the topic that have not even been touched on. The chapter [18] immediately following this one provides the reader with an excellent continuation of much of what has been set out in this introductory chapter. In addition, 3D photonic crystals have only been described by implication through the images in Figs. 1.1 and 1.2. The committed reader should look to the book by Joannopoulos and co-workers [6] for a more general understanding of the properties of photonic crystal structures that should suffice to support an odyssey into the research literature, whether the topic is full PBG 3D PhCs, PhC channel waveguides, PhC micro-cavities or photonic quasi-crystal light extraction structures for LEDs. Further recommended reading is exemplified in references [19–21].

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